Chapter 6
Passive Location of Emitter Source in Low Orbit Dual-Satellites System

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Abstract A kind of emitter TDOA (time difference of arrival, TDOA) and FDOA (frequency difference of arrival, FDOA) differential correction method based on GNSS signal was presented against the problem that the GNSS signal and the emitter source have same bias of TDOA and FDOA such as transponder delay and relative clock error. The TDOA correction and FDOA correction can be calculated in low orbit satellites through the information of GNSS direct signal and transponder signal and location information and velocity information of the navigation signal and low orbit satellites. Using the differential correction of TDOA and FDOA, the fixed bias of TDOA and FDOA can be best eliminated and then improved the position accuracy remarkably. This kind of method not only greatly reduced the dependence of the low orbit dual-satellites platform on the ground monitoring stations, but also increased the reliability, stability and precision. The simulation results show that this kind of method can easily achieve positioning accuracy better than 1 km in the entire coverage area.

Keywords Dual-satellites position · Emitter source · Differential · GNSS signal · Space-based

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6.1 Introduction

The precise positioning for emitter source has a special significance in the field of space information confrontation and satellite communication. Low orbit dual-satellites passive location received widespread attention with its characteristics of easy to be realized, less receive platforms and less resource occupation. Recently, the method of joint TDOA and FDOA is always used in low orbit passive position system. However, because of the influence of factors such as clock bias, transponder delay and fixed error of frequency, etc. the position error of which often reaches 3–10 km. As parameter measuring accuracy of time and frequency are huge factors which determined the location accuracy of emitter positioning, many scholars have researched and analyzed the dual-satellites passive location system and its location accuracy. Tim Pattison presented the linearized estimation problem arising from the dual-satellites geolocation of the source of a narrowband signal using TDOA and FDOA observations and an altitude constraint [1]. Wu Shilong analyzed and calculated the relationship between the positioning accuracy and dual-satellites positioning accuracy, and gave the dual-satellites system positioning accuracy performance and CEP curve in the conditions of different TDOA and FDOA [2, 3]. A kind of method using TDOA and FDOA was studied and presented which locates ground emitter from space satellites, the relationship between GDOP and TDOA error and FDOA error was also analyzed [4]. Recently, the studies on how to improve the position accuracy by eliminating the parameter measurement error were temporarily less in the literature at home and abroad.

The parameters of observation can be modified in GNSS differential location system and using the corrected measurement to position the source of emitter, high accuracy can be achieved. In view of differential position thought of satellite navigation system, by the mean time of using GNSS satellite signal to position the low orbit satellites, the system error of TDOA measuring and FDOA measuring can be greatly eliminated and the positioning accuracy can be remarkably improved by the use of low orbit dual-satellites differential positioning method. Direct against the situation of that the parameter measurement error influenced the positioning accuracy remarkably, low orbit dual-satellites emitter differential positioning system based on GNSS signal was proposed in this paper and by the use of differential technology, the measurement of TDOA and FDOA can be corrected with good effect. Simulation results show that the location accuracy can be improved remarkably using low orbit dual-satellites emitter differential positioning system based on GNSS signal auxiliary.

6.2 Low Orbit Dual-Satellites Differential Passive Location System

Through the measuring parameter arrived different observation platform, TDOA location technology format time difference Hyperboloid. Similarly, FDOA location technology format frequency difference Hyperboloid. If FDOA and TDOA are measured at the same time, we can get the intersection of time difference Hyperboloid and frequency difference Hyperboloid, and the target emitter's position can be calculated after fuzzy point removed. In the process of emitter dual-satellites location, the position accuracy was influenced by the aspects as follows:

1. Low orbit satellite determination error. The position and velocity of low orbit satellite was calculated through GNSS system instantly. While the two low orbit satellites was not far away from each other and the distance of which often reaches about 100 km, the GDOP of the two satellites in the GNSS navigation system can be considered equal and the low orbit satellite determination errors of two satellites are very close. The real sites of the low orbit satellites in the geocentric coordinate system is $\mathbf{S_i} = (x_i, y_i, z_i),\ i = 1, 2$, and the relative error of which is $\mathbf{dS_i} = (dx_i, dy_i, dz_i),\ i = 1, 2$, and the relationship between the broadcast satellite coordinate $\hat{\mathbf{S}}_\mathbf{i}$ and the real coordinate $\mathbf{S_i}$ is

$$\hat{\mathbf{S}}_\mathbf{i} = \mathbf{S_i} + \mathbf{dS_i} \tag{6.1}$$

   Where $i = 1, 2$.

   Supposed that the broadcast velocity is $\hat{\mathbf{v}}_\mathbf{i} = (\hat{v}_{xi}, \hat{v}_{yi}, \hat{v}_{zi}),\ i = 1, 2$, the relative velocity error is $\mathbf{dv_i}$, and the relationship between broadcast satellite velocity $\hat{\mathbf{v}}_\mathbf{i}$ and the true satellite velocity $\mathbf{v_i}$ is

$$\hat{\mathbf{v}}_\mathbf{i} = \mathbf{v_i} + \mathbf{dv_i} \tag{6.2}$$

   Where $i = 1, 2$.

2. The position deviation of the navigation satellites. The position deviation of the navigation satellites would affect the position accuracy of the dual-satellites system when the low orbit satellites was positioned through the parameter of the navigation satellites, in one way, it influenced the position accuracy of the low orbit satellites, and in another way, the position parameter of GEO satellites was used in low orbit dual-satellites emitter differential position system mentioned in this passage and the position error of the GEO satellites will bring some deviation to the system. The real site of the GEO satellite is $\mathbf{S_g} = (x_g, y_g, z_g)$, the broadcast site of the GEO satellite is $\hat{\mathbf{S}}_\mathbf{g} = (\hat{x}_g, \hat{y}_g, \hat{z}_g)$, the relative error of which is $\mathbf{dS_g} = (dx_g, dy_g, dz_g)$, and the relationship between $\hat{\mathbf{S}}_\mathbf{g}$ and $\mathbf{S_g}$ is

$$\hat{\mathbf{S}}_{\mathbf{g}} = \mathbf{S}_{\mathbf{g}} + \mathbf{dS}_{\mathbf{g}} \tag{6.3}$$

3. Relative clock error. In relative to the reference time, the satellites clock typically have a certain deviation which often reaches to 20 ns, with the relative clock error of the two low orbit satellites being reached up to 40 ns.
4. Transponder delay. The information communication between two low orbit satellites was implemented through the transponder and the delay of which maximum to 30 ns may caused by it.

Thus, we can draw to the conclusion that the main deviation source in the process of parameter measurement of low orbit dual-satellites emitter position system can be showed as follows

1. TDOA bias. The main TDOA bias source of the low orbit dual-satellites emitter position system include relative clock error, the transponder delay and the fixed bias caused by the ephemeris error. The random bias also makes an effect on the TDOA bias. So we can achieve

$$\Delta t = t_{TranD} + t_{Rclock} + t_{EPH} + \varepsilon_t \tag{6.4}$$

where Δt stands for the TDOA bias, t_{TranD} stands for transponder delay, t_{Rclock} is relative clock error, t_{EPH} is the fixed bias caused by the ephemeris error, and ε_t stand for the random TDOA bias.

2. FDOA bias. Because of the influence of the satellite velocity and the ephemeris error, fixed frequency bias was caused. The FDOA bias concludes the fixed frequency bias and the random FDOA bias. So

$$\Delta f = f_p + \varepsilon_f \tag{6.5}$$

Δf is FDOA bias, f_p is the fixed FDOA bias caused by satellite velocity and the ephemeris error. ε_f is the random FDOA bias.

In low orbit dual-satellites emitter position system, the positioning accuracy of which was influenced by satellite relative clock error, ephemeris error and ionosphere error etc. No matter where the locations of the users are, the influence of pseudo-range and carrier phase caused by satellite time difference error was similar. In a local radius, ephemeris error and ionosphere error have strong relevance, so we consider the bias caused by these errors to be equal. Thus, in the process of low orbit positioning, we can conclude that the position error of two low orbit satellites can be considered approximately equal in a uniform direction. A kind of low orbit dual-satellites emitter differential position system model was presented in this paper. This system consists of two low orbit satellites and the navigation satellites corresponded to them which include a GEO satellite. By the clever use of the navigation signal of GEO satellite, the fixed bias in the measurement of TDOA and FDOA can be eliminated effectively, then the position

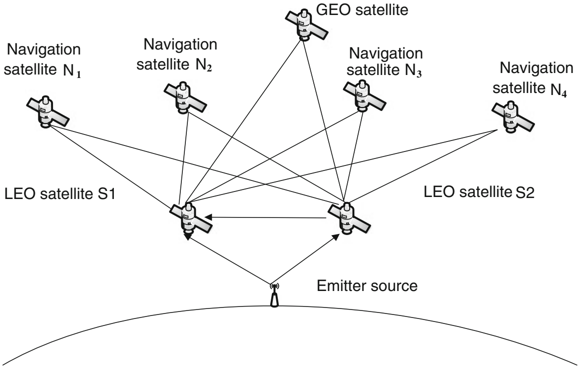


Fig. 6.1 Low orbit dual-satellites emitter differential position model

accuracy of which can be improved remarkably. This model based on space-based platform, and the low orbit satellite received GNSS signals and calculated the location and velocity of the low orbit satellites in the satellite instantly and the TDOA and FDOA of emitter signals can be corrected by the use of the parameter of the signal from the GEO satellite. The positioning model can be showed as follows (Fig. 6.1).

As the figure above shows, the GEO satellite signals can be transmitted to low orbit satellite $\mathbf{S_1}$ through two ways, the first one is from GEO satellite to low orbit satellite $\mathbf{S_1}$ directly, and the second one is from GEO satellite to low orbit satellite $\mathbf{S_2}$ firstly and then transmit it to the low orbit satellite $\mathbf{S_1}$. The total pseudo-range of the first way is

$$L_g^1 = d_g^1 + c^*(t_{Rclock1} + \varepsilon_{tg1}) \tag{6.6}$$

where $d_g^1 = \|\mathbf{S_g} - \mathbf{S_1}\|$, $t_{Rclock1}$ stands for the relative clock bias between satellite clock $\mathbf{S_1}$ and the reference time. ε_{tg1} stands for the random bias.

And the pseudo-range of the second way is

$$L_g^2 = d_g^2 + R + c^*(t_{TranD} + t_{Rclock2} + \varepsilon_{tg2}) \tag{6.7}$$

where $d_g^2 = \|\mathbf{S_g} - \mathbf{S}_2\|$, $R = \|\mathbf{S}_1 - \mathbf{S}_2\|$, $t_{Rclock2}$ stands for the relative clock bias between satellite clock $\mathbf{S_1}$ and the reference time. ε_{tg2} stands for the random bias.

The time difference of GEO satellite in the low orbit positioning platform is

$$\begin{aligned} dt_g &= (L_g^2 - L_g^1)/c \\ &= (d_g^2 + R - d_g^1)/c + t_{TranD} + t_{Rclock} + \varepsilon_{tg} \end{aligned} \tag{6.8}$$

Where $t_{Rclock} = t_{Rclock2} - t_{Rclock1}$ is the relative clock bias between two low orbit satellites. ε_{tg} is the comprehensive random time difference bias of GEO satellite.

We can get the TDOA correction by subtracting the distance calculated by the navigation information, and the main components of which includes the dual-satellites clock bias and the transponder delay, and they belong to the fixed deviation of the satellites positioning system, the time difference correction is

$$\begin{aligned} cdt &= \left(d_g^2 + R - d_g^1\right)/c - \left(\hat{d}_g^2 + \hat{R} - \hat{d}_g^1\right)/c \\ &\quad + t_{TranD} + t_{Rclock} + \varepsilon_{tt} \\ &\approx t_{TranD} + t_{Rclock} + \varepsilon_{tt} \end{aligned} \tag{6.9}$$

Where $\hat{d}_g^1 = \|\hat{\mathbf{S}}_\mathbf{g} - \hat{\mathbf{S}}_\mathbf{1}\|$, $\hat{d}_g^2 = \|\hat{\mathbf{S}}_\mathbf{g} - \hat{\mathbf{S}}_2\|$, $\hat{R} = \|\hat{\mathbf{S}}_1 - \hat{\mathbf{S}}_2\|$ The influence caused by the ephemeris error is very small when compared to the transponder delay and relative clock error.

The time difference of emitter source $\mathbf{u} = (x, y, z)$ in the low orbit satellites platform is

$$\begin{aligned} dt_f &= (L_f^2 - L_f^1)/c \\ &= (d_f^2 + R - d_f^1)/c + t_{TranD} + t_{Rclock} + \varepsilon_{tf} \end{aligned} \tag{6.10}$$

More precise TDOA can be obtained by subtracting the time difference correction and the distance between two satellites

$$\begin{aligned} \Delta t &= dt_f - cdt \\ &\approx \left(dt_f^2 - dt_f^1\right)/c + \varepsilon_t \end{aligned} \tag{6.11}$$

Similarly, the satellite signals will appear a Doppler shift in the process of space signal transmission, and because of the influence caused by the positioning error and the velocity error of the low orbit satellites etc., fixed deviation was formed which caused by the Doppler shift, so the Doppler shift frequency of GEO satellites signals can be shown as

$$f_g = f_{gt} + f_p + \varepsilon_{fg} \tag{6.12}$$

f_{gt} is the real FDOA of GEO satellites, f_p stands for the fixed frequency difference of GEO satellites caused by positioning error and the velocity error of the low orbit satellites.etc. ε_{fg} is the random frequency difference error.

We can calculated the frequency difference bias by the information of location and velocity of the satellites

$$\hat{f}_{gt} = -\frac{f_0}{c}\left[\frac{(\mathbf{S_g} - \mathbf{S}_2)^{\mathrm{T}}\mathrm{v}_2}{\|\mathbf{S_g} - \mathbf{S}_2\|} - \frac{(\mathbf{S_g} - \mathbf{S}_1)^{\mathrm{T}}\mathrm{v}_1}{\|\mathbf{S_g} - \mathbf{S}_1\|}\right] \tag{6.13}$$

The FDOA correction can be calculated by subtracting the calculated frequency difference bias

$$\begin{aligned} df &= f_{gt} - \hat{f}_{gt} + f_p + \varepsilon_{fg} \\ &\approx f_p + \varepsilon_{fg} \end{aligned} \tag{6.14}$$

And the FDOA of the emitter source in the low orbit dual-satellites differential platform can be shown as

$$f_f = f_{ft} + f_p + \varepsilon_{ff} \tag{6.15}$$

The more precise FDOA can be obtained by subtracting the frequency difference correction

$$\Delta f = f_{ft} + \varepsilon_f \tag{6.16}$$

According to the TDOA and FDOA after correction, simultaneous equations as follows

$$\Delta t = \frac{1}{c}\left(\|\mathbf{S_2} - \mathbf{u}\| - \|\mathbf{S}_1 - \mathbf{u}\|\right) \tag{6.17}$$

$$\Delta f = -\frac{f_0}{c}\left(\frac{(\mathbf{S_2} - \mathbf{u})^{\mathbf{T}}\mathbf{v_2}}{\|\mathbf{S_2} - \mathbf{u}\|} - \frac{(\mathbf{S}_1 - \mathbf{u})^{\mathbf{T}}\mathbf{v}_1}{\|\mathbf{S}_1 - \mathbf{u}\|}\right) \tag{6.18}$$

Assumed that the earth was ellipsoid model,

$$\mathbf{u}' = \left[\frac{x}{a_e}, \frac{y}{a_e}, \frac{z}{b}\right]^{\mathbf{T}} \tag{6.19}$$

Based on the equation of ellipsoid we can get

$$\mathbf{u}'^{\mathrm{T}}\mathbf{u}' = 1 \tag{6.20}$$

Where $a_e = 6378137$, $b = 6356752.314$, simultaneous Eqs. (6.17, 6.18 and 6.20), then we can get the emitter coordinates after calculating.

6.3 Performance Evaluation of Model

To seek partial differential to x, y, z respectively, we can get

$$\mathbf{H}\Delta\mathbf{u} = [\Delta t, \Lambda f, 0]^{\mathbf{T}} \tag{6.21}$$

Where $\mathbf{H} = [\mathbf{H_1}, \mathbf{H_2}, \mathbf{H_3}]^{\mathbf{T}}$, $\Delta\mathbf{u} = (dx, dy, dz)^{\mathbf{T}}$

$$\mathbf{H_1} = \frac{1}{c}\left(\frac{(\mathbf{u} - \mathbf{S_2})^{\mathbf{T}}}{\|\mathbf{S_2} - \mathbf{u}\|} - \frac{(\mathbf{u} - \mathbf{S_1})^{\mathbf{T}}}{\|\mathbf{S_2} - \mathbf{u}\|}\right) \tag{6.22}$$

$$\mathbf{H_2} = \frac{f_0}{c}\left(-\frac{(\mathbf{u} - \mathbf{S_2})^{\mathbf{T}}\mathbf{V_2}(\mathbf{u} - \mathbf{S_2})}{\|\mathbf{u} - \mathbf{S_2}\|^2} + \frac{\mathbf{V}_2^{\mathbf{T}}}{\|\mathbf{u} - \mathbf{S_2}\|} + \frac{(\mathbf{u} - \mathbf{S_1})^{\mathbf{T}}\mathbf{V_1}(\mathbf{u} - \mathbf{S_1})}{\|\mathbf{u} - \mathbf{S_1}\|^2} - \frac{\mathbf{V}_1^{\mathbf{T}}}{\|\mathbf{u} - \mathbf{S_1}\|}\right) \tag{6.23}$$

$$\mathbf{H_3} = \left[\frac{2x}{a_e^2}, \frac{2y}{a_e^2}, \frac{2z}{b^2}\right] \tag{6.24}$$

Thus, the covariance matrix of positioning error is

$$\psi = E\left\{\Delta\mathbf{u}^T \Delta\mathbf{u}\right\} = \mathbf{H}^{-1} diag\left\{\sigma_t^2, \sigma_f^2, 0\right\}\mathbf{H}^{-\mathbf{T}} \tag{6.25}$$

The GDOP of the positioning error is

$$GDOP\,(x, y, z) = \sqrt{\psi\,(1, 1) + \psi\,(2, 2) + \psi\,(3, 3)} \tag{6.26}$$

Generally, the blind zone will be formed in the link line of the two low orbit satellites in low orbit dual-satellites passive location system. In the blind zone, the GDOP will tend towards infinity. Thus, we need to avoid the blind zone or use other ways to improve the position accuracy in this kind of area. In this paper, the improvement of low orbit dual-satellites passive location accuracy emphasize on that taking method to eliminate the fixed bias of TDOA and FDOA, and issue of principle, properties of low orbit dual-satellites passive location and blind zone, interested readers can consult relative literature for detail.

6.4 Analysis of Simulation Result

6.4.1 Analysis of Position Accuracy in the Middle Line of Sub-satellite Points

Supposed that the longitude and latitude of the two satellites are (125, 29.4), (125, 30.5), and 1100 km above the earth. The velocities of the two low orbit satellites are approximately equal to (743.8, 3228.6, 6503.2), and the distance between two satellites are 143 km, the relative clock error of dual-satellites are $t_{Rclock} = 40\,\text{ns}$ and the transponder delay $t_{TranD} = 30\,\text{ns}$. Considered the random error and skipped the blind zone, the comparison of position error distribution in the middle line of sub-satellite point can be shown as follows (Figs. 6.2, 6.3).

The result of the simulation shows that the position accuracy of low orbit dual-satellites emitter differential system is improved largely when compared to the formal dual-satellites location system. In the blind zone which in the middle line of sub-satellite point, the position errors of which are very largely and tend towards infinity. And in the two sides of the blind zone, the performance of position error is good. With the rapid growth of the distance in the two sides, the position errors turn to be bigger. Form the figure we can get that the position error before differential in the middle line of sub-satellite point turned to be 1–3 km, and after differential, the position error of which down to less than 100 m. This can reflects the position error influenced by the distance in some extent.

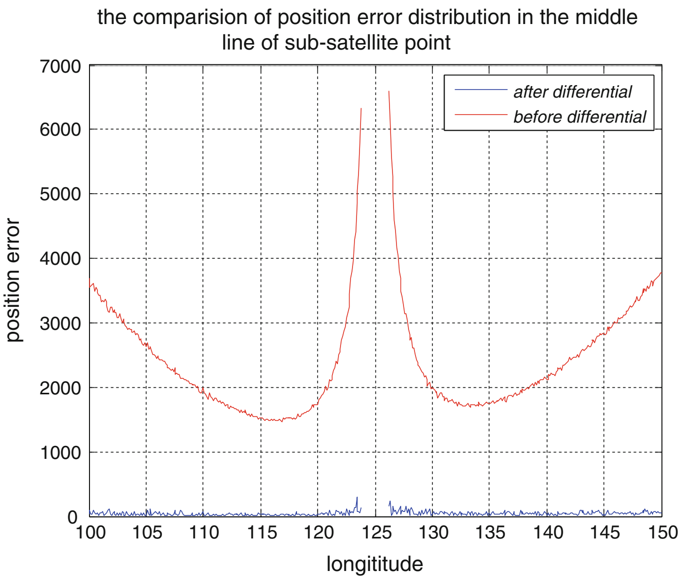


Fig. 6.2 Position error comparison in the *middle line* of sub-satellite point before differential and after differential

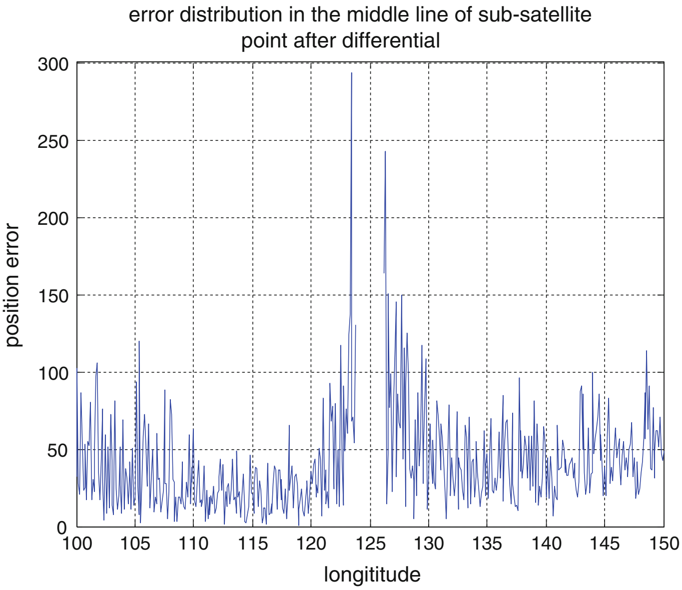


Fig. 6.3 Position error in the *middle line* of sub-satellite point after differential

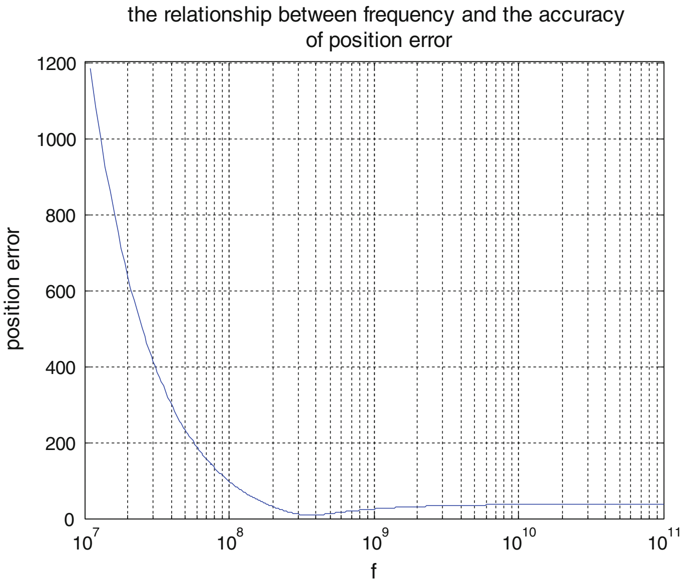


Fig. 6.4 The relationship between frequency and the position error

6.4.2 Position Error Influenced by the Frequency

In low orbit dual-satellites emitter differential position system, the frequency of GEO satellite signals $f_g = 1.5 * 10^9$ constantly, while the frequencies of ground emitter are uncertain. The simulation results of the position accuracy of different emitter frequency in the same condition can be shown as follows (Fig. 6.4).

We can learn that position errors are rather big when the frequency magnitude is in 10^7, which is greater than 1 km. When the frequency magnitude is in 10^8, the position error turn to be smaller, but in some band of frequencies, the position error is still great and cannot be neglected. When the frequencies magnitudes are in 10^9 and 10^{10}, the position errors are small and steadily, with a slight increase. When the frequency of the emitter is lower, the influence of fixed frequency bias is rather big and it will bring big influence to the position accuracy. But when the frequency of emitter source turned to reach a certain level, the influence caused by the emitter source in the calculated GEO frequency bias will increase, so it will influence the FDOA correction and make the position error a little bigger. We can get the best position accuracy in the magnitude of 10^9 and 10^{10}.

Fig. 6.5 The target diagram of the comparison

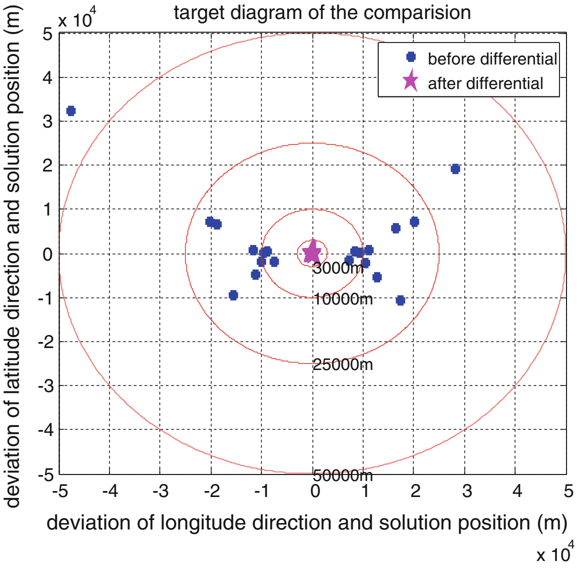
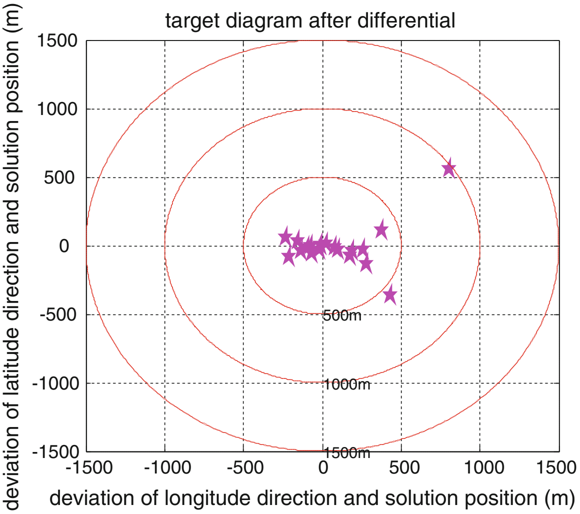


Fig. 6.6 The target diagram after differential



6.4.3 *Simulation Effect of Position Error*

Take 20 groups of coordinates in the position area, and we can get the target diagram of the correction as follows (Figs. 6.5, 6.6).

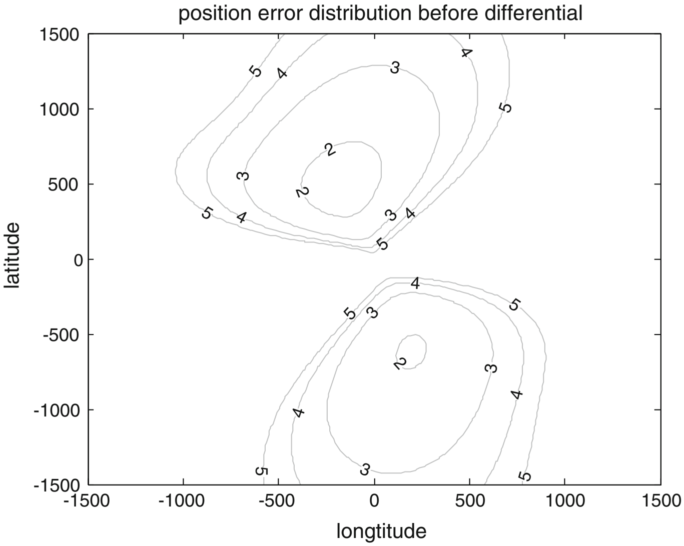


Fig. 6.7 Position error distribution before differential

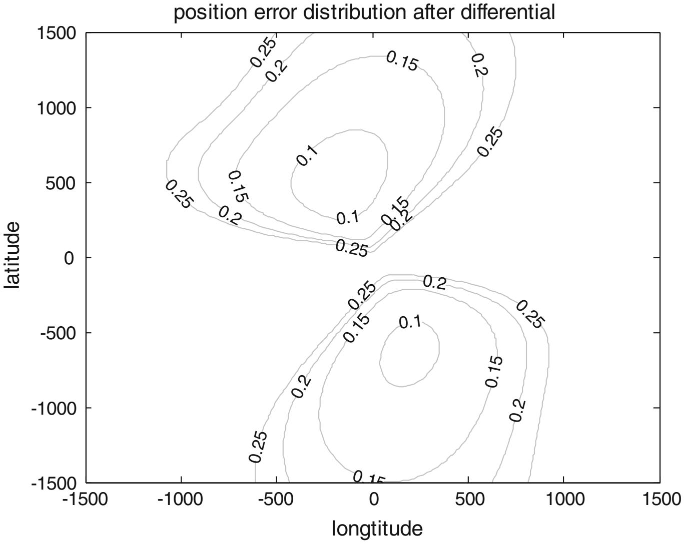


Fig. 6.8 Position error distribution before differential

From the target diagram we can come to conclusion that the position accuracy of the low orbit dual-satellites emitter position system is low, which ranges 3–10 km, and in some area the position error even surpass 10 km. But after differential, the position error of which control to 500 m. From the simulation result we can get that the position accuracy of which can be improved remarkably with differential, and the position area turn to be larger when limiting the range of position error.

6.4.4 The Comparison of Position Error Distribution Before Differential and After Differential

We compared the GDOP of low orbit dual-satellites emitter position before differential and after differential as follows (Figs. 6.7, 6.8).

From the simulation results above we draw a conclusion that the position accuracy after differential are better than the one before differential and in the same position area the position error after differential can depress to under 1 km while the traditional system came to 3–10 km. The position accuracy can be improved remarkably using the low orbit dual-satellites emitter differential position system based on GNSS signal auxiliary, and possess applicable value.

6.5 Conclusions

In this paper, the source of position error in the low orbit dual-satellites location system was comprehensive analyzed and a kind of low orbit dual-satellites location algorithm based on GNSS differential principle was proposed. This method based on space-based platform and the dependence of the low orbit dual-satellites platform on the ground monitoring stations was reduced through the orbit determination and TDOA and FDOA correction in the low orbit satellites, and with the less dependence on the ground station, the reliability and the stability when the ground station get disturbed turned to be improved. In view of differential position thought of satellite navigation system, the fixed bias of TDOA and FDOA can be best eliminated, thus improved the passive location accuracy. Simulation results show that by the use of this kind of method, position accuracy can be easily achieved better than 1 km when compared to 3–10 km in traditional way.

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